

Assignment 2

Coverage: 15.2, 15.3 in Text.

Exercises: 15.2. no 23, 25, 27, 31, 35, 39, 55, 57, 61, 65, 69, 71, 75, 77, 79. 15.3. no 5, 7, 12, 15, 18, 29, 30.

Submit 15.2 no. 61; 15.3 no 15, 18 by Sept 28.

Supplementary Problems

1. Let S be a non-empty set in \mathbb{R}^n . Define its characteristic function χ_S to be $\chi_S(\mathbf{x}) = 1$ for $\mathbf{x} \in S$ and $\chi_S(\mathbf{x}) = 0$ otherwise. Prove the following identities:

(a) $\chi_{A \cup B} \leq \chi_A + \chi_B$.

(b) $\chi_{A \cup B} = \chi_A + \chi_B$ if and only if $A \cap B = \phi$, that is, A and B are disjoint.

(c) $\chi_{A \cap B} = \chi_A \chi_B$.

2. Let f be integrable in a region D which satisfies $A \leq f \leq B$ for two numbers A and B everywhere. Show that

$$A|D| \leq \int_D f \leq B|D|,$$

where $|D|$ is the area of D .